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Superposed Coherent and Two Laser Light Beams

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Abstract
In this paper, we have studied the squeezing and statistical properties of superposed three light beams. In order to carry out the analysis, we have obtained the superposition density operator along with the Q-functions, we have calculated the mean photon number, the photon number variance and the quadrature variance for the superposed three light beams. It has been found that the mean photon number and quadrature variances of the superposed three light beams are the sum of the mean photon numbers and quadrature variances of the separate light beams, respectively. The quadrature squeezing of the superposed three light beams is the average quadrature squeezing of the three light beams. In addition, the mean photon number and quadrature squeezing of the superposed three light beams increase with linear gain coefficient.
Keywords: Superposed, Squeezing, Density operator.

1. Introduction

Quantum properties of the light generated by various optical systems such as lasers and with the effect of light on the dynamics of atoms. The quantum properties of light are largely determined by the state of the light mode. The quantum states of light are number, chaotic, coherent and squeezed states [1-6]. In a squeezed state the quantum noise in one quadrature is below the vacuum level at the expense of enhanced fluctuations in the conjugate quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation. Squeezing is a nonclassical feature of light. Squeezed light has potential applications in the detection of weak signals and in low-noise communications [1-6].

A three-level laser is quantum optical system in which three level atoms in a cascade configuration, initially prepared in a coherent superposition of the top and bottom levels, are injected into a cavity coupled to a vacuum reservoir via a single-port mirror. When a three-level atom in cascade configuration, it makes a transition from the top to the bottom level via the intermediate level, two photons are generated. If the generated light modes have the same frequency, the three-level laser is said to be a degenerate three-level laser; otherwise it is called a nondegenerate three-level laser [1, 7].

Some authors have arrived at the conclusion that the superposition of coherent light beam with some other light beam does not affect the quadrature variance of the other light beam [8 -10]. Fesseha has studied the statistical and squeezing properties of superposed coherent and squeezed light produced by one mode subharmonic generations in the same cavity. Applying a slightly modified definition for the quadrature variance of a pair of superposed light beams and comparing with the quadrature variance of a single coherent light beam. He has shown the quadrature squeezing of the superposed light beams is half of the squeezed light. This is just the average quadrature squeezing of the separate light beams [11].

In this paper, we seek to study the squeezing and statistical properties of the superposed three light beams produced by a coherent light and a pair of degenerate three-level lasers. In order to carry out the analysis, we first obtain the density operator for the superposed three light beams in terms of the respective Q-functions.

2. Density Operator

Suppose $\hat{\rho}'(\hat{a}_1^+, \hat{a}_1, t)$ is the density operator for the first light beam, say for the coherent light beam. Then upon expanding this density operator in normal order and applying the completeness relation for coherent state, we see that

$$\hat{\rho}'(\hat{a}_1^+, \hat{a}_1, t) = \int d^2\alpha_1 Q_1(\alpha_1^*, \alpha_1 + \frac{\partial}{\partial \alpha_1^*}, t) \hat{D}(\alpha_1) \hat{\rho}(0) \hat{D}(-\alpha_1), \tag{1}$$

where

$$Q_1(\alpha_1^*, \alpha_1 + \frac{\partial}{\partial \alpha_1^*}, t) = \frac{1}{\pi} \sum_{ij} C_{ij} \alpha_1^{*i} (\alpha_1 + \frac{\partial}{\partial \alpha_1^*})^j, \tag{2}$$

is the Q-function associated with the coherent light beam.

On the bases of Eq. (1), the density operator for superposition of the second light beam with the first one and for superposition of the third light beam with the first as well as the second light beams can be written, respectively as

$$\hat{\rho}^{''}(\hat{a}_2^+,\hat{a}_2,t)=\int d^2\alpha_2 Q_2(\alpha_2^*,\alpha_2+\frac{\partial}{\partial\alpha_2^*},t)\hat{D}(\alpha_2)\hat{\rho}^{'}(t)\hat{D}(-\alpha_2), \tag{3}$$

$$\hat{\rho}(\hat{a}^+,\hat{a},t)=\int d^2\alpha_3 Q_3(\alpha_3^*,\alpha_3+\frac{\partial}{\partial\alpha_3^*},t)\hat{D}(\alpha_3)\hat{\rho}^{''}(t)\hat{D}(-\alpha_3), \tag{4}$$

in which

$$Q_2(\alpha_2^*,\alpha_2+\frac{\partial}{\partial\alpha_2^*},t)=\frac{1}{\pi}\sum_{kl}C_{kl}\alpha_2^{*k}(\alpha_2+\frac{\partial}{\partial\alpha_2^*})^l, \tag{5}$$

$$Q_3(\alpha_3^*,\alpha_3+\frac{\partial}{\partial\alpha_3^*},t)=\frac{1}{\pi}\sum_{mn}C_{mn}\alpha_3^{*m}(\alpha_3+\frac{\partial}{\partial\alpha_3^*})^n, \tag{6}$$

is the Q-function associated with the second and third light beam respectively. Now combination of Eqs. (1), (3), and (4) then using Baker-Hausdorff identity and the definition that the coherent state is the displacement operator acting on a vacuum state, we easily obtain

$$\hat{\rho}(\hat{a}^+,\hat{a},t)=\int d^2\alpha_1 d^2\alpha_2 d^2\alpha_3 Q_1(\alpha_1^*,\alpha_1+\frac{\partial}{\partial\alpha_1^*},t)Q_2(\alpha_2^*,\alpha_2+\frac{\partial}{\partial\alpha_2^*},t)$$
$$\times Q_3(\alpha_3^*,\alpha_3+\frac{\partial}{\partial\alpha_3^*},t)\left|\alpha_1+\alpha_2+\alpha_3\right\rangle\left\langle\alpha_3+\alpha_2+\alpha_1\right|. \tag{7}$$

Eq. (7) represents the density operator for superposition of the three light beams.

3. Photon Statistics

In this section we seek to study the statistical properties of a superposed three light beams. We calculate the mean and variance of the photon number for the superposed three light beams employing the resulting density operator and the Q-functions of the respective light beams.

3.1 The Mean Photon Number

The mean photon number can be expressed in terms of the density operator as [15]

$$\bar{n}_s = Tr(\hat{\rho}(t)\hat{a}^+\hat{a}), \tag{8}$$

Where the subscript "s" stands for the superposed beams. Applying Eq.(7)in Eq.(8), we get

$$\bar{n}_s = <\hat{a}_1^+(t)\hat{a}_1(t)> + <\hat{a}_2^+(t)\hat{a}_2(t)> + <\hat{a}_3^+(t)\hat{a}_3(t)> + <\hat{a}_2^+(t)><\hat{a}_3(t)> + <\hat{a}_2(t)><\hat{a}_3^+(t)>$$
$$+ <\hat{a}_2^+(t)><\hat{a}_1(t)> + <\hat{a}_1^+(t)><\hat{a}_3(t)> + <\hat{a}_1(t)><\hat{a}_3^+(t)> + <\hat{a}_1^+(t)><\hat{a}_2(t)>$$

(9)

in which

$$<\hat{a}_i^+(t)\hat{a}_i(t)>=\int d^2\alpha_i Q_i(\alpha_i^*,\alpha_i+\frac{\partial}{\partial\alpha_i^*},t)\alpha_i^*\alpha_i, i=1,2.3. \tag{10}$$

We next proceed to determine the expectation values $<\hat{a}_1(t)>$,we can write as

$$<a_1(t)>=\int d^2\alpha_1 Q_1(\alpha_1^*,\alpha_1+\frac{\partial}{\partial\alpha_1^*},t)\alpha_1, \tag{11}$$

in which

$$Q_1(\alpha_1^*,\alpha_1+\frac{\partial}{\partial\alpha_1^*},t)=Q_1(\alpha_1^*,\alpha_1,t)\exp[(q(t)-\alpha_1^*)\frac{\partial}{\partial\alpha_1^*}], \tag{12}$$

with

$$Q_1(\alpha_1^*,\alpha_1,t)=\frac{1}{\pi}\exp[-\alpha_1^*\alpha_1+q(t)(\alpha_1+\alpha_1^*)-q^2(t)], \tag{13}$$

is the Q-function for the coherent light. Where $q(t)=\frac{2\varepsilon}{k}(1-e^{-kt/2})$, using Eq. (12) in Eq.(11), we can be put in the form

$$< a_1(t) >= \frac{\partial}{\partial \lambda}\int d^2\alpha_1 [Q_1(\alpha_1^*, \alpha_1, t) D(\lambda, \alpha^*{}_1, \alpha_1)]|_{\lambda=0}, \tag{14}$$

where

$$D(\lambda, \alpha^*{}_1, \alpha_1) = \exp[(q(t) - \alpha_1^*)\frac{\partial}{\partial \alpha_1^*}]\exp(\lambda\alpha_1). \tag{15}$$

Based on eigenvalue equation for a differential operator $\hat{A}$:

$$\hat{A}f(x) = af(x), \tag{16}$$

that satisfies

$$e^{\hat{A}} f(x) = e^a f(x) \tag{17}$$

We easily get

$$D(\lambda, \alpha^*{}_1, \alpha_1) = \exp(\lambda\alpha_1). \tag{18}$$

Substituting Eqs. (18) and (13) into Eq.(14), then carrying out the integration and upon performing the differentiation and also setting λ = 0, we get

$$< \hat{a}_1(t) >= q(t). \tag{19}$$

Following a similar procedure, one can easily verify that

$$< \hat{a}_2(t) >=< \hat{a}_3(t) >=< \hat{a}_2^+(t) >=< \hat{a}_3^+(t) >= 0, \tag{20}$$

$$< \hat{a}_1^+(t)\hat{a}_1(t) >= q^2(t), \tag{21}$$

$$< \hat{a}_2^+(t)\hat{a}_2(t) >= \frac{u_1}{u_1{}^2 - v_1{}^2} - 1 = a_1 - 1, \tag{22}$$

and

$$< \hat{a}_3^+(t)\hat{a}_3(t) >= \frac{u_2}{u_2{}^2 - v_2{}^2} - 1 = a_2 - 1, \tag{23}$$

in which $\quad u_2 = \frac{a_2}{a_2{}^2 - b_2{}^2}, v_2 = \frac{b_2}{a_2{}^2 - b_2{}^2},$

$$a_2 = 1 + \frac{A_1(1-\eta)}{2(A_1\eta + k)}(1 - e^{-(A_1\eta+k)t}, b_2 = \frac{A_1\sqrt{1-\eta^2}}{2(A_1\eta + k)}(1 - e^{-(A_1\eta+k)t} \quad . \tag{24}$$

Finally, the mean photon number of the superposed three light beams is

$$\overline{n}_s = \frac{4\varepsilon^2}{k^2}(1 - e^{-kt/2})^2 + \frac{A_1(1-\eta)}{2(A_1\eta + k)}(1 - e^{-(A_1\eta+k)t}) + \frac{A_2(1-\eta)}{2(A_2\eta + k)}(1 - e^{-(A_2\eta+k)t}). \tag{25}$$

At steady state the mean photon number turns out to be

$$\overline{n}_{sss} = \frac{4\varepsilon^2}{k^2} + \frac{A_1(1-\eta)}{2(A_1\eta + k)} + \frac{A_2(1-\eta)}{2(A_2\eta + k)}. \tag{26}$$

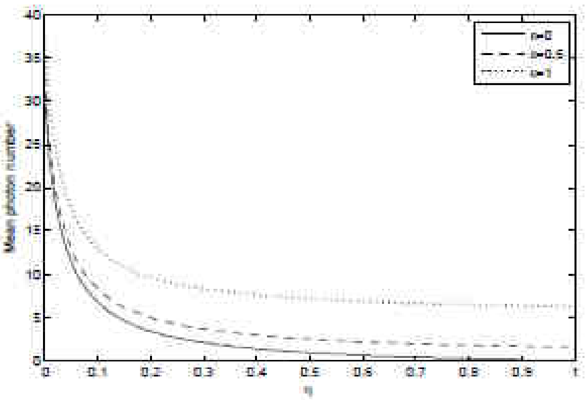

Fig.1: Plots of the mean photon number [Eq. (26)] versus η for k = 0.8, A = 25, and for different values of ε.

Fig. 1 represents the mean photon number [Eq. (426)] versus η, for k = 0.8, A = 25, and for ε = 0(solid curve),

ε = 0.5(dashed curve), and ε = 1(dotted curve). The figure indicates that the mean photon number increases with ε and decreases with η.

We observe from Eq. (26) that the mean photon number of the superposed three light beams is the sum of the mean photon numbers of a coherent light and a pair of degenerate three-level lasers.

We consider the case in which the two lasers have the same linear coefficient, upon setting $A_1 = A_2 = A$, Eq. (26) reduces to

$$\bar{n}_{sss} = \frac{4\varepsilon^2}{k^2} + \frac{A(1-\eta)}{A\eta + k}. \tag{27}$$

We easily see from Eq. (27) that the mean photon number of the superposed three light beams is the mean photon number of a coherent light plus two times of the mean photon number of degenerate three-level laser.

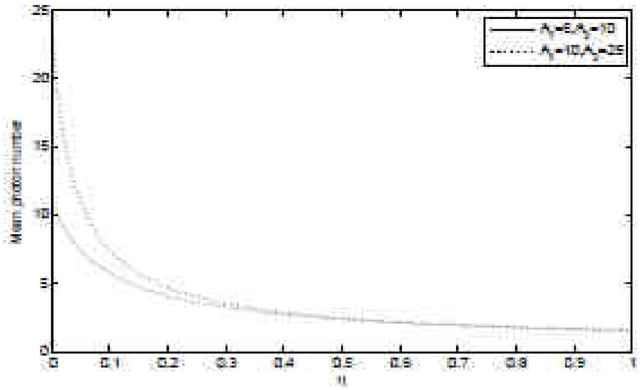

Fig. 2: Plots of the mean photon number [Eq. (427)] versus η for k = 0.8, ε = 0.5, and for different values liner gain coefficients (A_1,A_2).

The plots in Fig. 2 represent the mean photon number [Eq. (427)] versus η, for k = 0.8, ε = 0.5, and for A_1= 5,A_2 = 10 (solid curve), and A_1 = 10,A_2 = 25 (dotted curve). The figure shows that the mean photon number increase with the liner gain coefficients (A_1, A_2) and decreases with η.

3.2 The Variance of Photon Number

The variance of the photon number for the superposed three light beams is defined by

$$\Delta n^2{}_s = < (\hat{a}^+(t)\hat{a}(t))^2 > - \bar{n}_s^2. \tag{28}$$

Using commutation relation

$$\left[\hat{a}, \hat{a}^+\right] = 3, \tag{29}$$

where

$$\hat{a} = \hat{a}_1 + \hat{a}_2 + \hat{a}_3, \tag{30}$$

with the aid of Eq. (29), Eq. (28) can be written in the form

$$\Delta n^2{}_s = < \hat{a}^{+2}\hat{a}^2 > - \bar{n}_s^2 + 3\bar{n}_s. \tag{31}$$

Therefore, the variance of the photon number for the superposed three light beams turns out to

$$\Delta n^2{}_s = \bar{n}_2^2 + \bar{n}_3{}^2 + 2\bar{n}_1(b_1 + b_2) + (b_1 + b_2)^2 + 2\bar{n}_1(\bar{n}_2 + \bar{n}_3) + 2\bar{n}_2\bar{n}_3 + 3\bar{n}_{s.} \tag{32}$$

We easily see from Eq. (32) that the variance of photon number for superposed three light beams is greater than that of the mean photon number. This shows that the photon statistics of the superposed three light beams is super Poissonian. The variance of photon number for superposed three light beams is does not the sum of the separate light beams.

4. QuadratureVariance

In this section we seek to determine the quadrature variance of the superposed three light beams. The quadrature operator for the superposed three light beams is defined by

$$\hat{a}_\pm = \sqrt{\pm 1}(\hat{a}^+ \pm \hat{a}), \tag{33}$$

with the modified commutation relation

$$[\hat{a}_{+,}\hat{a}_-] = 6i. \tag{34}$$

In view of this commutation relation, superposed three light beams is said to be in a squeezed state if either $\Delta a_+ < 3$ or $\Delta a_- < 3$ such that $\Delta a_+ \Delta a_- \geq 3$. The quadrature variances can be defined by

$$\Delta \hat{a}_\pm^2 = < \hat{a}_\pm^2 > - < \hat{a}_\pm >^2 . \tag{35}$$

Using Eq.(29)and (33), we can write as

$$\Delta \hat{a}_\pm^2 = 3 + 2 < \hat{a}^+ \hat{a} > \pm < \hat{a}^2 > \pm < \hat{a}^{+2} > \mp < \hat{a}^+ >^2 \mp < \hat{a} >^2 - 2 < \hat{a}^+ >< \hat{a} > . \tag{36}$$

Finally, the quadrature variance of the superposed three light beams is turns to be

$$\Delta \hat{a}_\pm^2 = 3 + \frac{A_1[(1-\eta) \pm \sqrt{1-\eta^2}\,]}{A_1\eta + k}(1 - e^{-(A_1\eta + k)t}) + \frac{A_2[(1-\eta) \pm \sqrt{1-\eta^2}\,]}{A_2\eta + k}(1 - e^{-(A_2\eta + k)t}). \tag{37}$$

We observe that the quadrature variance for superposed three light beams is the sum of the quadrature variances of the separate light beams. We consider a special case, in which the two lasers have the same linear gain coefficient. Hence upon setting $A_1 = A_2 = A$, we get

$$\Delta \hat{a}_\pm^2 = 3 + \frac{2A[(1-\eta) \pm \sqrt{1-\eta^2}\,]}{A\eta + k}(1 - e^{-(A\eta + k)t}). \tag{38}$$

Upon setting t = 0, the quadrature variance Eq. (37) reduces to

$$(\Delta \hat{a}_\pm^2)_c = 3 \tag{39}$$

Eq. (39) represents the quadrature variances for the three mode coherent or vacuum state.

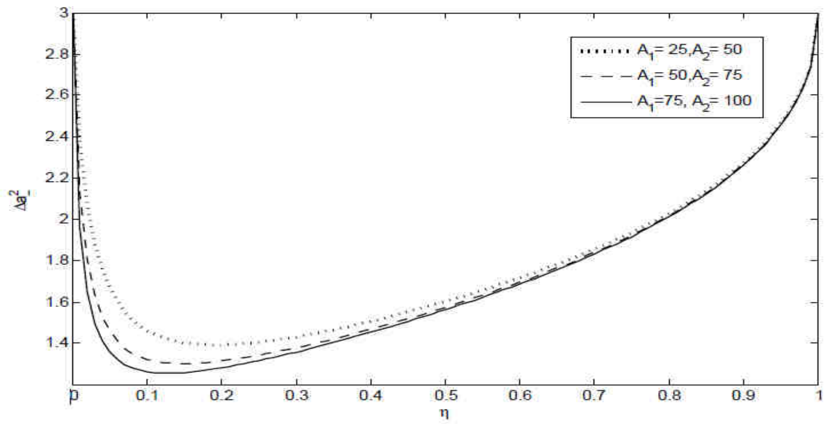

Fig.3: Plots of the quadrature variance $\Delta \hat{a}_-^2$ [Eq.(37)]versus η for k = 0.8 and for different values of the linear gain coefficients(A_1,A_2).

Fig.3 represents the variance of the minus quadrature Eq. (37) versus η , for k = 0.8 and for $A_1 = 25$,$A_2 = 50$(dotted curve), $A_1 = 50$,$A_2 = 75$(dashed curve),and $A_1 = 75$,$A_2 = 100$(solid curve). We observe from the figure that the quadrature squeezing increases with the linear gain coefficients. Moreover, the maximum quadrature squeezing described by Eq. (37) for $A_1 = 75$,$A_2 = 100$ and k = 0.8, is found to be 58.13% and occurs at η = 0.12 below the coherent state level.

5. Conclusion

In this paper, we have studied the squeezing and statistical properties the superposition of a coherent light and a pair of degenerate three-level lasers. In order to carry out the analysis, we have obtained the superposition density operator along with the Q-functions, we have calculated the mean photon number, variance of the photon number, and the quadrature variance for the superposed three light beams. It is found that the mean photon number of superposed three light beams is the sum of the mean photon number of the separately light beams and quadrature variances of superposed three light beams is the sum of the quadrature variances of the separately light beams. In addition, the mean photon number of the superposed three light beams increase with linear gain coefficient and proportional to amplitude of driving coherent light. It is also found that the quadrature squeezing of the superposed three light beams is the average values of the three light beams. This results shows that the presence of the coherent light decreases the quadrature squeezing of the superposed three light beams. Moreover, for $A_1 = 75$, $A_2 = 100$, k = 0.8, the maximum squeezing for superposed three light beams is found to be 58.13% and occurs at η = 0.12 below the coherent state level.

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